Parallel Computing of Action Potentials in the Hodgkin-Huxley Model via the Parareal Algorithm

Eric Boerman, Khanh Pham, Katie Peltier

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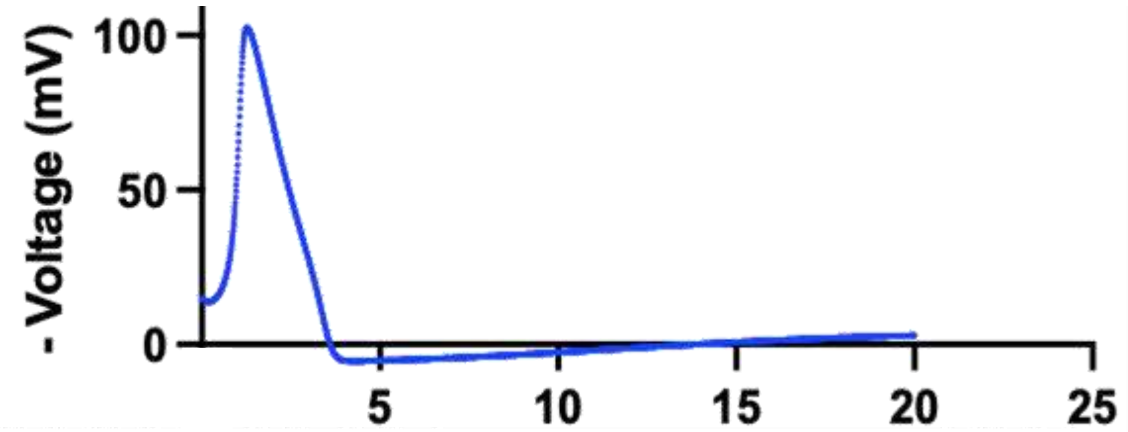
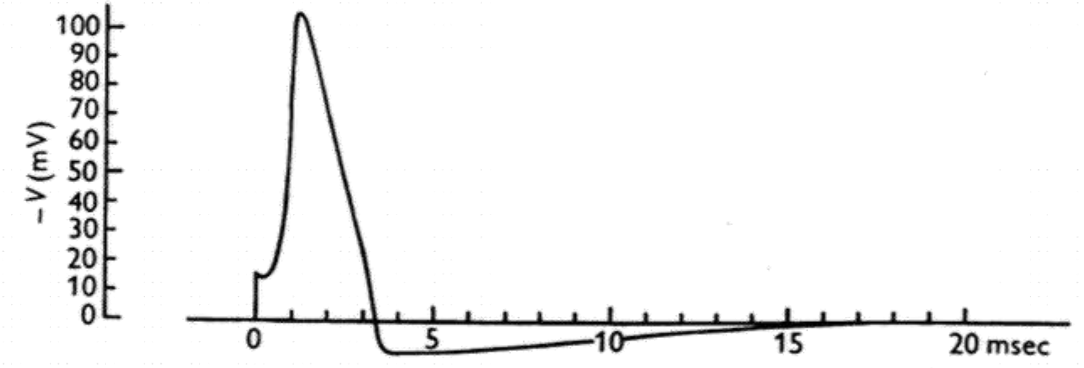
The Action Potential



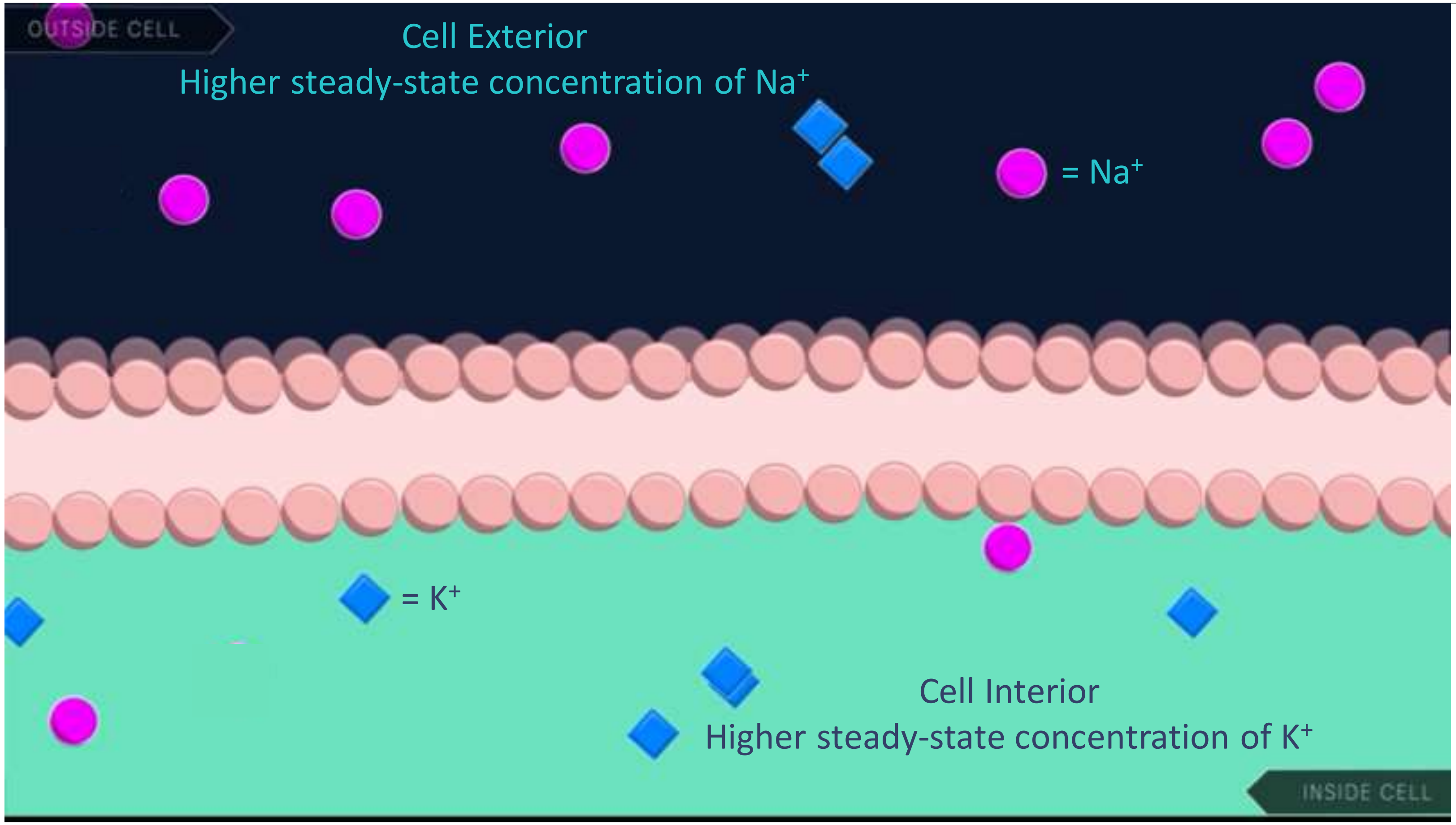


Source: *A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve,* Hodgkin & Huxley, 1952

Above: Numerical approximation of an action potential in the Hodgkin-Huxley model

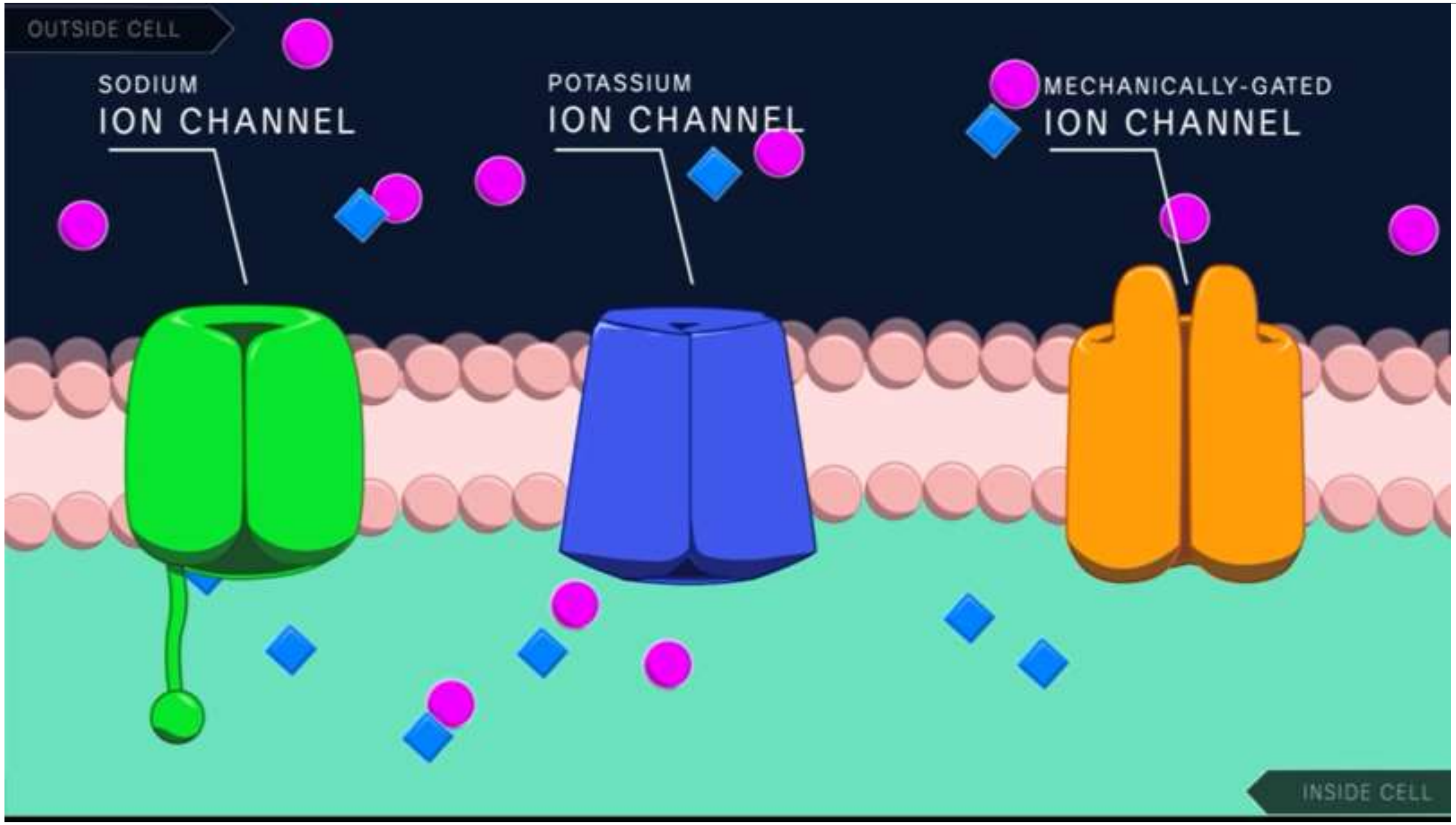
Below: Numerical solution as reported by Hodgkin and Huxley in 1952

The Cell Membrane



Source: Action Potential in the Neuron, Harvard Extension School. https://www.youtube.com/watch?v=oa6rvUJlg7o

The Cell Membrane



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The Hodgkin-Huxley Model

Ohm's Law – current equals voltage times conductance

Total current is the sum of all component currents:

$I = I_1 + I_2 + \cdots + I_n$

For each ionic current, I_{ion} = conductance (g_{ion}) times distance from voltage equilibrium:

$I = g_K(V - V_k) + g_{Na}(V - V_{Na}) + g_l(V - V_l) + C_m \frac{dv}{dt}$

where $C_m \frac{dv}{dt}$ is the current from the membrane's function as a capacitor

The Hodgkin-Huxley Model

Conductance for Na^+ and K^+ (g_{Na} and g_K) are gated by voltage

n, m, and h are proportions (0 ≤ n, m, h ≤ 1) that vary with voltage and define gate activation or inactivation

$\bar{g}_{Na}$ and $\bar{g}_K$ are the maximum possible conductances for a given set of parameters

$$g_K = \bar{g}_{Na} n^4$$

$$g_{Na} = \bar{g}_K m^3 h$$

g_l does not meaningfully vary with voltage, and is treated as constant

The Hodgkin-Huxley Model

$$\frac{dv}{dt} = (I - \bar{g}_K n^4(V - V_k) - \bar{g}_{Na} m^3 h(V - V_{Na}) - g_l(V - V_l))/Cm$$

$$\frac{dm}{dt} = \alpha_m(1 - m) - \beta_m m$$

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n$$

$$\frac{dh}{dt} = \alpha_h(1 - h) - \beta_h h$$

i	α_i	β_i
m	$\frac{2.5 + 0.1V}{e^{2.5+0.1V} - 1}$	$4e^{\frac{V}{18}}$
n	$\frac{0.01V + 0.1}{e^{0.1V+1} - 1}$	$0.125e^{\frac{V}{80}}$
h	$0.07e^{\frac{V}{20}}$	$\frac{1}{1 + e^{3+0.1V}}$

Numerical Methods

Forward Euler Method

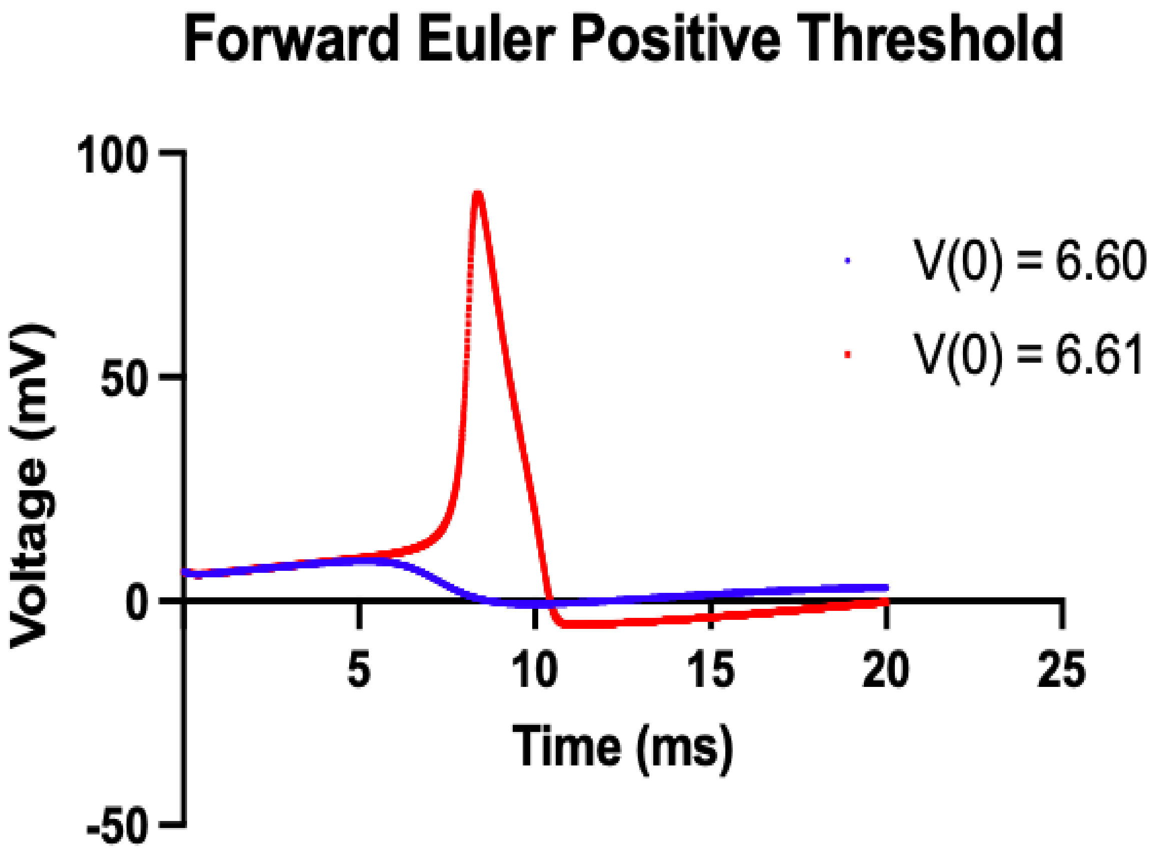
- Fastest numerical method
- Relatively inaccurate: 1^{st}-order accuracy

4^{th}-Order Runge-Kutta Method

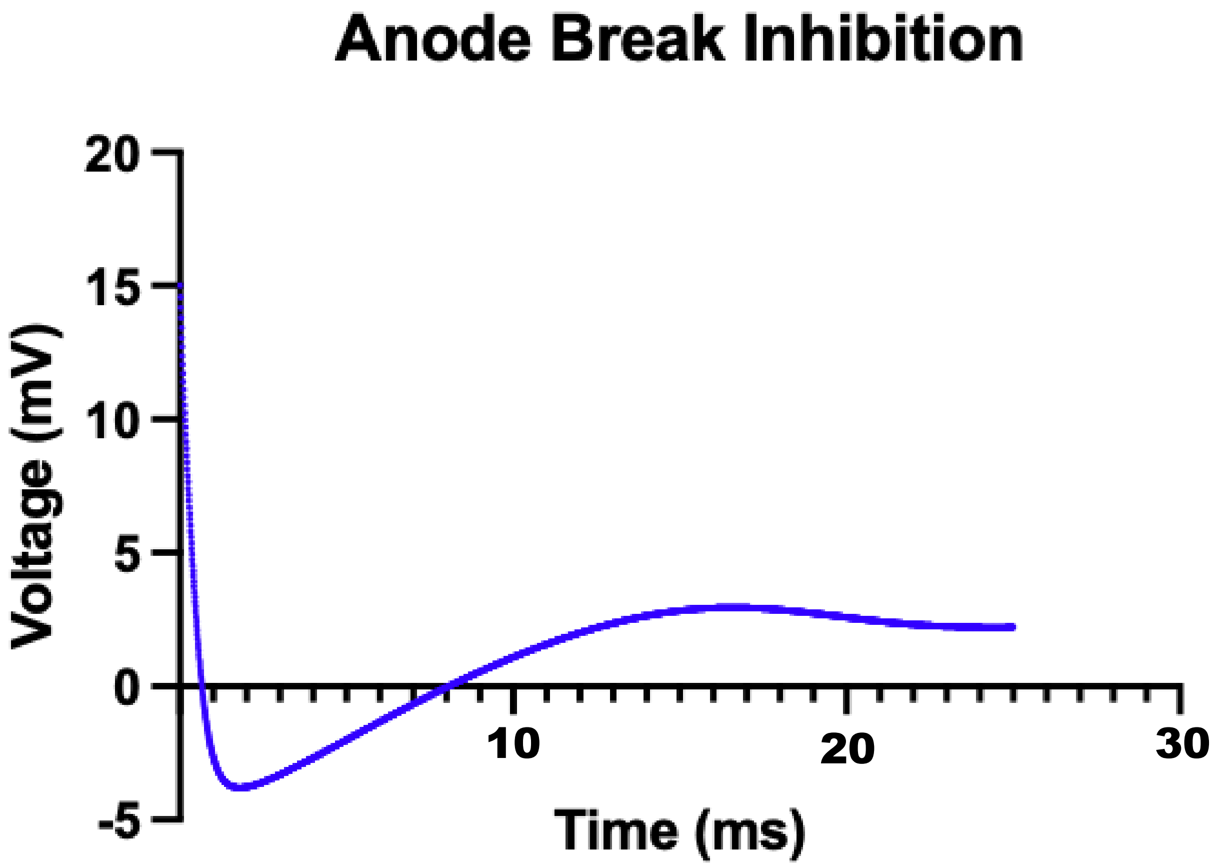
- Increased accuracy given same parameters
- Computationally more expensive

For both methods, V, n, m, and h are solved for simultaneously within each step.

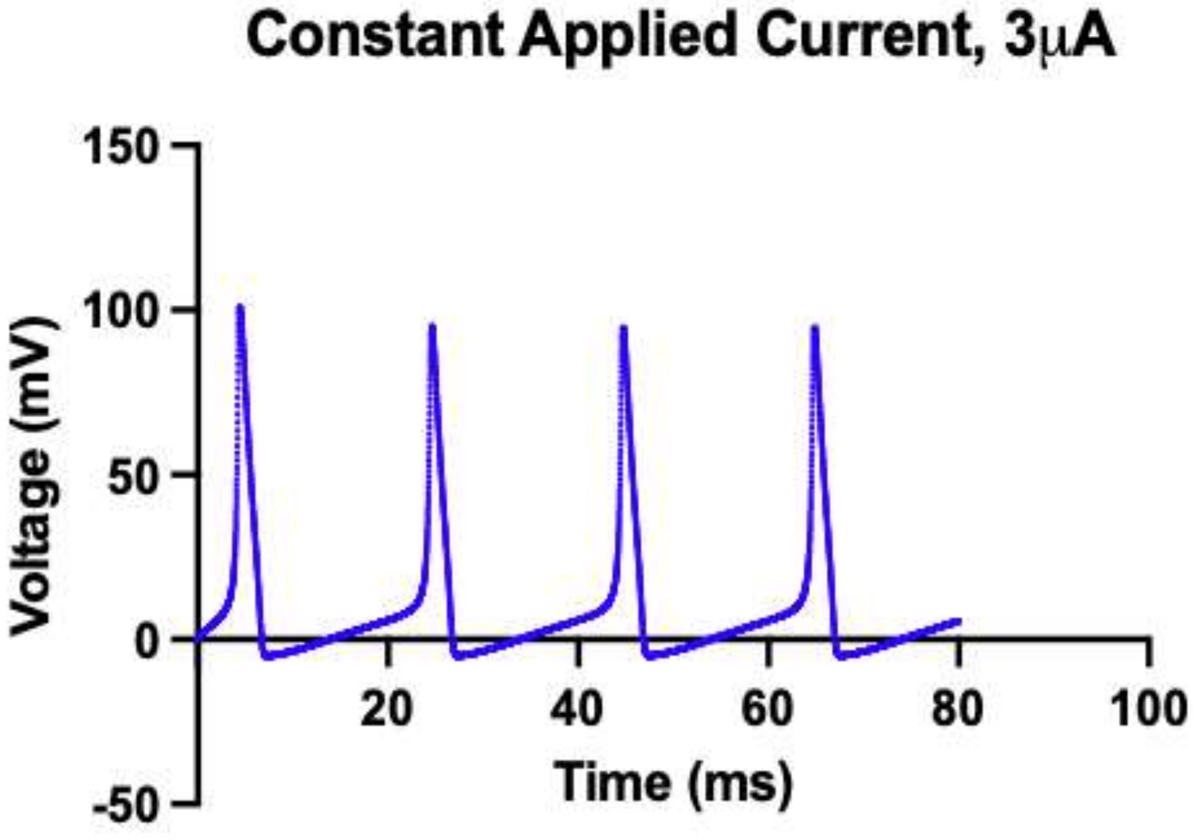
Experimental Results



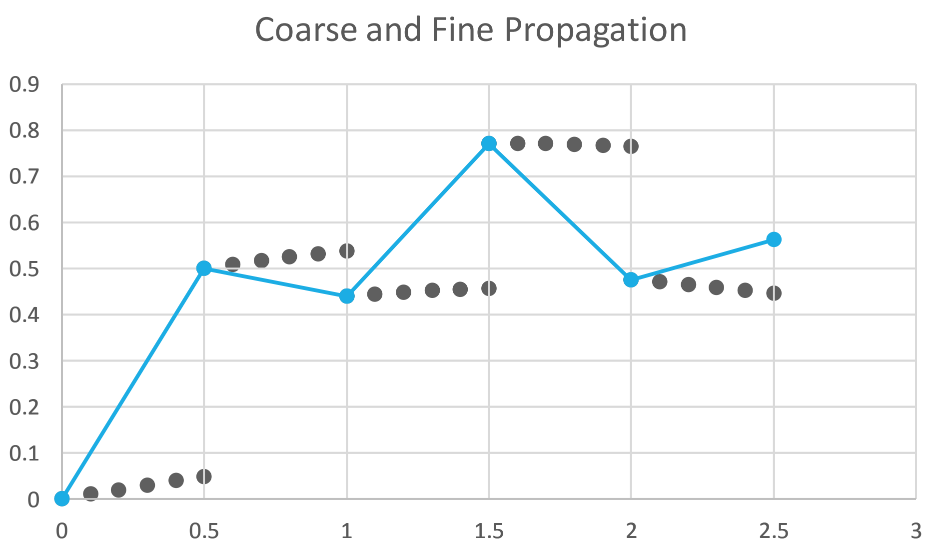
Experimental Results



Experimental Results



Parareal



- A unique parallel-in-time algorithm, developed by Lions, Meday, and Turinici in 2001
- Utilizes two temporal discretizations – one coarse, one fine – and solves them numerically
- Predicts reasonable starting values, then calculates fine mesh values in parallel
- Converges to a solution over multiple iterations
- Does not increase accuracy over sequential method, but can offer significant time savings

Parareal

Tolerance (mV)	Iterations (47 max)
10^-5	3
10^-6	4
10^-7	4
10^-8	5

Preliminary estimations for parallelization in a 48-CPU system suggest a significant possible decrease in computational time

At 47 iterations time savings is negative compared to sequential calculations, but the Parareal algorithm finishes well before then, even for tolerances within 1/100,000,000th of a millivolt

At increased CPU counts (100, 200, etc.), iteration count seems to fall around ~5% of maximum at this tolerance level

While computational overhead limits maximum possible time savings, preliminary results suggest that for most real-world scenarios increasing the CPU count will increase efficiency

References

Hodgkin, A L, and A F Huxley. “A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve.” *Bulletin of Mathematical Biology*, vol. 52, no. 1-2, 1990, pp. 500–544., doi:10.1016/s0092-8240(05)80004-7.

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Questions?