McKibben Webster – Chapter 3 Partial Solutions and Hints

3.1.1 $$\begin{cases} \dfrac{dT}{dt} = k_1\left(T(t) - T_m(t)\right) \\ \dfrac{dT_m}{dt} = k_2 T_m(t) + k_3 T(t) \\ T(0) = T_0,\ T_m(0) = T_1 \end{cases}$$

3.1.2 (i) Expect $|k_3| < |k_1| < |k_2|$. Why?

(ii) $$\begin{cases} \dfrac{dT_R}{dt} = k_1\left(T_R(t) + T_p(t) - T_m(t)\right) \\ \dfrac{dT_p}{dt} = k_2\left(T_R(t) + T_p(t) - T_m(t)\right) \\ \dfrac{dT_m}{dt} = k_3 T_m(t) + k_4 T_R(t) + k_5 T_p(t) \\ T_R(0) = T_0,\ T_p(0) = T_1,\ T_m(0) = T_2 \end{cases}$$

3.1.3 (i) $$\begin{cases} \dfrac{dT}{dt} = k_1\left(T(t) - T_m(t)\right) \\ \dfrac{dT_0}{dt} = k_3 T_0(t) \\ \dfrac{dT_m}{dt} = k_3 T_m(t) + k_4 T_0(t) \\ T(0) = T_0,\ T_0(0) = T_1,\ T_m(0) = T_2 \end{cases}$$

where T_0 is the temperature of the area.

3.2.1 (i) Change the initial amount present in the soft tissue

(ii) $U_{ST}(0) = u_1 + w$

3.2.2 (i) Draw a diagram

(ii) $$\begin{cases} \dfrac{dU_{ST}}{dt} = a_{H,ST}U_H - a_{ST,H}U_{ST} \\ \dfrac{dU_{ST}}{dt} = \left(a_{ST,H}U_{ST} + a_{L,H}U_L\right) - \left(a_{H,ST}U_H + a_{H,E}U_H - a_{H,GI}U_H\right) \\ \dfrac{dU_L}{dt} = \left(a_{P,L}U_P + a_{H,L}U_H + a_{GI,L}U_{GI}\right) - a_{L,H}U_L \\ \dfrac{dU_P}{dt} = a_{H,P}U_H - a_{P,L}U_P \\ \dfrac{dU_E}{dt} = a_{H,E}U_H - LU_E \\ \dfrac{dU_{GI}}{dt} = a_{H,GI}U_H - a_{GI,L}U_{GI} \\ \text{Usual initial conditions for each of the six compartments} \end{cases}$$

3.3.1 Let $v_1(t)$ and $v_2(t)$ be the volumes of tanks 1 and 2, respectively.

$$\begin{cases} \dfrac{dU_{T_1}(t)}{dt} = \dfrac{2U_{T_2}(t)}{v_2(t)} - \dfrac{7U_{T_1}(t)}{v_1(t)} - \dfrac{l_1U_{T_1}(t)}{v_1(t)} \\ \dfrac{dU_{T_2}(t)}{dt} = \dfrac{7U_{T_1}(t)}{v_1(t)} - \dfrac{2U_{T_2}(t)}{v_2(t)} - \dfrac{5U_{T_2}(t)}{v_2(t)} \\ \dfrac{dv_1(t)}{dt} = -l_1 \\ \dfrac{dv_2(t)}{dt} = -l_2 \\ U_{T_1}(0) = 20,\ U_{T_2}(0) = 30 \\ v_1(0) = 100,\ v_2(0) = 80 \end{cases}$$

3.4.1 (i) If pacifist, then there is no grievance and so $g_j = 0$. Also, pacifist nations are probably not allocating resources to train soldiers or buying weapons, so that $k_j = 0$.

(ii) $u(t) = \begin{bmatrix} e^{-c_1t} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-c_nt} \end{bmatrix} u_0$

(iii) All nations downsize their forces toward zero.

3.4.2 $$\begin{cases} \dfrac{du_1}{dt} = -c_1u_1 + k_3u_3 \\ \dfrac{du_2}{dt} = -c_2u_2 + k_3u_3 \\ \dfrac{du_3}{dt} = -c_3u_3 + g_1 + g_2 \\ \text{plus initial conditions for each } u_i \end{cases}$$

3.5.1 Two – prescribe initial position of the mass and the initial speed

3.5.2 The forces would need to change. Tell why and how.

3.6.1 Two each – they represent the initial concentrations of A and B

3.11.1 (i) $$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} T(t) \\ T_m(t) \end{bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} T(t) \\ T_m(t) \end{bmatrix} \\ \begin{bmatrix} T(0) \\ T_m(0) \end{bmatrix} = \begin{bmatrix} T_0 \\ T_1 \end{bmatrix} \end{cases}$$

(ii) $$\begin{cases} \dfrac{d}{dt}\begin{bmatrix} y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} -a & 0 \\ a & -b \end{bmatrix}\begin{bmatrix} y(t) \\ z(t) \end{bmatrix} \\ \begin{bmatrix} y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} y_0 \\ 0 \end{bmatrix} \end{cases}$$