# McKibben Webster Chapter 9 – Partial Solutions and Hints

## MatLab – Exercise 9.1.1

- i) Everything seems to be running properly. However, the help button doesn't seem to be working
- ii) Still seems to be running fine.
- iii) Comparing the two figures, it looks like i is faster than ii
- iv) It seem that the larger k^2 is, the easier it is for the heat to disperse
- v) My conjecture seems to be correct.
- vi) Nuemann boundary conditions
  - a. Runs fine
  - b. Still runs fine
  - c. Comparing the two figures, they look like they are similar in speed
  - d. The troughs in the waves seem shallower when k^2 is larger
  - e. My conjecture seems to be correct
- vii) Summary the equation seems to depend on k^2, because the larger k^2 is, the more diluted the equation will be.
- viii) Periodic boundary conditions
  - a. Runs fine
  - b. Still runs fine
  - c. Comparing the two figures, they look like they are similar in speed
  - d. The troughs in the waves seem shallower when k^2 is larger
  - e. My conjecture seems to be correct

### Matlab-Exercise 9.1.2

- i) Runs fine, converges
- ii) Runs fine, less convergence than k^2=.1
- iii) Cos initial condition
  - a. Converges to an arc
  - b. Less convergence than  $k^2=.25$
  - c. Convergence seems to depend on k^2
- iv) Step functions  $K^2 = .25$  converges when  $k^2$  is larger, does seem to depend on  $k^2$

v) The larger the value of  $k^2$ , the greater the convergence

### Matlab-Exercise 9.1.3

- i) sin
- a. 0.5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.4994
- b. Repeat (a)
  - i. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.2497
  - ii. 0.1- Norm Difference vs. Time, Difference in parameters = 0Norm Difference between Initial Conditions = 0.9988
  - iii. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.009988
  - iv. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.004994
- c. Doesn't seem to depend on the initial condition
- d. Exp and step function
  - i. Exp
    - 1. .5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.48794
    - 2. Repeat (1)
      - a. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.24397
      - b. 0.1- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.097588
      - c. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.0097588
      - d. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.0048794
    - 3. Doesn't seem to depend on initial conditions
  - ii. Step

1. .5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.5

- 2. Repeat (1)
  - a. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.25
  - b. 0.1- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.1
  - c. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.01
  - d. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.005
- 3. Doesn't seem to depend on initial conditions

Note: for this part I am using perturbation size .5

- ii) Cos
  - a. 0.4- Norm Difference vs. Time, Difference in parameters = 0.1 Norm Difference between Initial Conditions = 0.99039
  - b. Repeat (a)

- i. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.99039
- ii. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.99039
- iii. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 Norm Difference between Initial Conditions = 0.99039
- iv. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0.99093
- c. Doesn't seem to depend on initial conditions
- d. Exp and step function
  - i. Exp

1. .4- Norm Difference vs. Time, Difference in parameters = 0.1 Norm Difference between Initial Conditions = 0.48794

- 2. Repeat (1)
  - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 48794
  - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 48794
  - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 Norm Difference between Initial Conditions = 0. 48794
  - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0.48794
- 3. Doesn't seem to depend on initial conditions

### ii. Step

1. .4- Norm Difference vs. Time, Difference in parameters = 0.1 Norm Difference between Initial Conditions = 0.5

- 2. Repeat (1)
  - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.25
  - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.1
  - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 Norm Difference between Initial Conditions = 0.01
  - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0.005
- e. Doesn't seem to depend on initial conditions

iii) Exp

a. Diffusivity constant: 0.4- Norm Difference vs. Time, Difference in parameters = 0.1
 Perturbation size: 0.3- Norm Difference between Initial Conditions = 0.29276

- b. Repeat (a)
  - i. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0.14638
  - ii. 0.55- Norm Difference vs. Time, Difference in parameters = 0.050.1- Norm Difference between Initial Conditions = 0.097588
  - iii. 0.48- Norm Difference vs. Time, Difference in parameters = 0.020.01- Norm Difference between Initial Conditions = 0.0097588
  - iv. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 0.005- Norm Difference between Initial Conditions = 0.0048794

- c. Doesn't seem to depend on initial conditions
- d. Cos and step function
  - i. Cos
    - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1
      - .3- Norm Difference between Initial Conditions = 0.59424
    - 2. Repeat (1)
      - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0. 29712
      - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05
        0.1- Norm Difference between Initial Conditions = 0.19808
      - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02
        0.01- Norm Difference between Initial Conditions = 0. 019808
      - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 0.005-Norm Difference between Initial Conditions = 0.0099039
    - 3. Doesn't seem to depend on initial conditions
  - ii. Step
    - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1
      - .3- Norm Difference between Initial Conditions = 0.3
    - 2. Repeat (1)
      - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0.15
      - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05
        0.1- Norm Difference between Initial Conditions = 0.1
      - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02
        0.01- Norm Difference between Initial Conditions = 0.01
      - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 0.005-Norm Difference between Initial Conditions = 0.005
- e. Doesn't seem to depend on initial conditions
- iv) Not the case

### Matlab-Exercise 9.1.4

Note: L<sup>2</sup> button is actually L2

- i) sin
- a. 0.5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.5
- b. Repeat (a)
  - i. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.25
  - ii. 0.1- Norm Difference vs. Time, Difference in parameters = 0Norm Difference between Initial Conditions = 0.1
  - iii. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.01
  - iv. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.005
- c. Doesn't seem to depend on the initial condition
- d. Exp and step function
  - i. Exp

- 1. .5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.17701
- 2. Repeat (1)
  - a. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.088505
  - b. 0.1- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.035402
  - c. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.0035402
  - d. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.0017701
- 3. Doesn't seem to depend on initial conditions

#### ii. Step

- 1. .5- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.5
- 2. Repeat (1)
  - a. .25- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.25
  - b. 0.1- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.1
  - c. 0.01- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.01
  - d. 0.005- Norm Difference vs. Time, Difference in parameters = 0 Norm Difference between Initial Conditions = 0.005
- 3. Doesn't seem to depend on initial conditions

Note: for this part I am using perturbation size .5

- v) Cos
  - a. 0.4- Norm Difference vs. Time, Difference in parameters = 0.1
    - Norm Difference between Initial Conditions = 0.86603
  - b. Repeat (a)
    - i. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 86603
    - ii. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 86603
    - iii. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02Norm Difference between Initial Conditions = 0. 86603
    - iv. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0. 86603
  - c. Doesn't seem to depend on initial conditions
  - d. Exp and step function
    - i. Exp
      - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1 Norm Difference between Initial Conditions = 0.17701
      - 2. Repeat (1)
        - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 17701
        - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0. 17701

- c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 Norm Difference between Initial Conditions = 0. 17701
- d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0. 17701
- 3. Doesn't seem to depend on initial conditions
- ii. Step
  - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1 Norm Difference between Initial Conditions = 0.5
  - 2. Repeat (1)
    - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.5
    - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05 Norm Difference between Initial Conditions = 0.5
    - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 Norm Difference between Initial Conditions = 0.5
    - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 Norm Difference between Initial Conditions = 0.5
- e. Doesn't seem to depend on initial conditions

vi) Exp

- a. Diffusivity constant: 0.4- Norm Difference vs. Time, Difference in parameters = 0.1
  Perturbation size: 0.3- Norm Difference between Initial Conditions = 0.10621
- b. Repeat (a)
  - i. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0.053103
  - ii. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05
    0.1- Norm Difference between Initial Conditions = 0.035402
  - iii. 0.48- Norm Difference vs. Time, Difference in parameters = 0.020.01- Norm Difference between Initial Conditions = 0.0035402
  - iv. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 0.005- Norm Difference between Initial Conditions = 0.0017701
- c. Doesn't seem to depend on initial conditions
- d. Cos and step function
  - i. Cos
    - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1
      - .3- Norm Difference between Initial Conditions = 0.51926
    - 2. Repeat (1)
      - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0. 25981
      - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05
        0.1- Norm Difference between Initial Conditions = 0.17321
      - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02 0.01- Norm Difference between Initial Conditions = 0.017321
      - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01
        0.005-Norm Difference between Initial Conditions = 0.0086603
    - 3. Doesn't seem to depend on initial conditions
  - ii. Step
    - 1. .4- Norm Difference vs. Time, Difference in parameters = 0.1
      - .3- Norm Difference between Initial Conditions = 0.3

- 2. Repeat (1)
  - a. .45- Norm Difference vs. Time, Difference in parameters = 0.05 .15- Norm Difference between Initial Conditions = 0.15
  - b. 0.55- Norm Difference vs. Time, Difference in parameters = 0.05
    0.1- Norm Difference between Initial Conditions = 0.1
  - c. 0.48- Norm Difference vs. Time, Difference in parameters = 0.02
    0.01- Norm Difference between Initial Conditions = 0.01
  - d. 0.51- Norm Difference vs. Time, Difference in parameters = 0.01 0.005-Norm Difference between Initial Conditions = 0.005

## e. Doesn't seem to depend on initial conditions

ii. There is continuous dependence.

## Matlab-Exercise 9.1.5

Note: Boundary Condition set to Dirichlet

- i. Sin
- a. It seems lopsided at the beginning this could be because the nonhomogeneous part off sets things to one side
- b. Nonhomogeneousness flattens out on a higher plane
- c.  $T_1 = .5, T_2 = 2$ 
  - i. It just seems to converge to a convex line
  - ii. Homogeneousness flattens out, but nonhomogeneousness just seems to slope up

### ii. Exp

- a. Nonhomogeneousness arks upward
- b. Homogeneousness has a spike that flattens out, while nonhomogeneousness has a spike that flattens into a prabolla in the opposite direction
- c.  $T_1 = .5, T_2 = 2$ 
  - i.  $T_1$  off sets the left side down and  $T_2$  off sets the right side up
  - ii. Homogeneousness flattens out while nonhomogeneousness slopes up

### iii.

- a. Yes, it does seem that (i) tends to a steady-state temperature for large times
- b. Homogeneousness seems to tend to a stedy-state temperature much earlier than nonhomogeneousness
- c. Yes, it does seem that (ii) tends to a steady-state temperature for large times

d. Homogeneousness seems to tend to a stedy-state temperature earlier than nonhomogeneousness

## MatLab-Exercise 9.1.6

- i) Dirichlet
  - a. Runs fine
  - b. Runs fine
  - c. The heat seems to disperse much more slowly
  - d. It seems that the lower k^2 is, the slower the dispersion is. Which is the same as 9.1
  - e. The speed of the dispersion seems to be positively correlated to k^2

## ii) Periodic

- a. Runs fine
- b. Runs fine
- c. Seems to run faster with higher k^2
- d. The higher  $k^2$  is, the faster the heat dispersion, the same as 9.1
- e. The higher k^2 is the faster the heat dispersion

## iii) Neumann

- a. Runs fine
- b. Runs fine
- c. Seems to run faster with higher k^2
- d. The higher  $k^2$  is, the faster the heat dispersion, the same as 9.1
- e. The higher k^2 is the faster the heat dispersion
- iv) The higher k^2 is, the faster the heat dispersion regardless of the boundary condition

## MatLab-Exercise 9.1.7

- i) Runs fine
  - a. Converges to the same function
- ii) Runs fine
  - a. Yes the solution with BC periodic converged to the same function as (i)
- iii) The bottom rises as the top falls
- iv) They all converged to the same function. However, they all converged differently

## MatLab-Exercise 9.1.8

Note: Perturb Parameters & Solve is actually Modify parameters & solve

- i) Note: BC is Dirichet and k<sup>2</sup> = .5
  - a. 0.5 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.5
  - b. Repeat
    - i. 0.25 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.25
    - ii. 0.1 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.1
    - iii. 0.01 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.01
    - iv. 0.005 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.005
  - c. It doesn't seem to depend on the diffusivity constant
  - d. Expo and step
    - i. Expo
      - 0.5 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.099083
      - 2. Repeat
        - a. 0.25 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.049542
        - b. 0.1 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.019817
        - c. 0.01 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.0019817
        - d. 0.005 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.00099086
      - 3. It seems to depend on the diffusivity constant
    - ii. Step

1. 0.5 - Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.5

- 2. Repeat
  - a. 0.25 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.25
  - b. 0.1 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.1
  - c. 0.01 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.01
  - d. 0.005 Norm Difference vs. Time Difference in Parameters = 0 Norm Difference between Initial Conditions = 0.005
- 3. It seems to depend on the diffusivity constant
- ii) BC Neumann, initial condition Cos

- a. 0.5 Norm Difference vs. Time Difference in Parameters = 0.1 Norm Difference between Initial Conditions = 1.4142
- b. Repeat
  - i. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 1.4142
  - ii. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 1.4142
  - iii. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02 Norm Difference between Initial Conditions = 1.4142
  - iv. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01 Norm Difference between Initial Conditions = 1.4142
- c. It seems to depend on the diffusivity constant
- d. Expo and step
  - і. Ехро
    - 1. 0.4 Norm Difference vs. Time Difference in Parameters = 0.1 Norm Difference between Initial Conditions = 0.099083
    - 2. Repeat
      - a. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 0.099083
      - b. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 0.099086
      - c. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02 Norm Difference between Initial Conditions = 0.099086
      - d. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01 Norm Difference between Initial Conditions = 0.099083
    - 3. It seems to depend entirely on the diffusivity constant
  - ii. Step
    - 1. 0.4 Norm Difference vs. Time Difference in Parameters = 0.1 Norm Difference between Initial Conditions = 0.5
    - 2. Repeat
      - a. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 0.25
      - b. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05 Norm Difference between Initial Conditions = 0.1
      - c. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02 Norm Difference between Initial Conditions = 0.01
      - d. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01 Norm Difference between Initial Conditions = 0.005
    - 3. It seems to depend on the diffusivity constant
- iii) BC Periodic, initial condition expo
  - a. 0.4 Norm Difference vs. Time Difference in Parameters = 0.1
    - 0.3 Norm Difference between Initial Conditions = 3.8048
  - b. Repeat
    - i. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05 0.25 - Norm Difference between Initial Conditions = 3.1707
    - ii. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05
      - 0.1 Norm Difference between Initial Conditions = 1.2683
    - iii. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02

- 0.01 Norm Difference between Initial Conditions = 0.1283
- iv. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01
  0.005 Norm Difference between Initial Conditions = 0.063413
- c. It doesn't seem to depend on the diffusivity constant
- d. Cos and step
  - i. Cos
    - 1. 0.4 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.3 Norm Difference between Initial Conditions = 54.3058
    - 2. Repeat
      - a. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05 0.25 - Norm Difference between Initial Conditions = 45.2548
      - b. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05
        0.1 Norm Difference between Initial Conditions = 18.1019
      - c. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02
        - 0.01 Norm Difference between Initial Conditions = 1.8102
      - d. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01
        0.005 Norm Difference between Initial Conditions = 0.9051
    - 3. It seems to depend heavily on the perturbation size
    - ii. Step
      - 1. 0.4 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.3 Norm Difference between Initial Conditions = 18.6
      - 2. Repeat
        - a. 0.45 Norm Difference vs. Time Difference in Parameters = 0.05
          0.25 Norm Difference between Initial Conditions = 15.5
        - b. 0.55 Norm Difference vs. Time Difference in Parameters = 0.05
          0.1 Norm Difference between Initial Conditions = 6.2
        - c. 0.48 Norm Difference vs. Time Difference in Parameters = 0.02
          0.01 Norm Difference between Initial Conditions = 0.62
        - d. 0.51 Norm Difference vs. Time Difference in Parameters = 0.01
          0.005 Norm Difference between Initial Conditions = 0.31
      - 3. It seems to depend on the perturbation size
- iv) In 2d the difference between the initial conditions, and the perturbations is much greater than in 1d

### MatLab-Exercise 9.2.1

- i) Heat was less wavy
  - a. Convergence happens much faster
  - b. The smaller  $\alpha$  is, the slower the convergence seems to be
  - c. No convergence, possible stability
  - d. The lower  $\beta$  is, the slower convergence seems to be
  - e. Expo

- i. Convergence happens much faster
- ii. The smaller  $\alpha$  is, the slower the convergence seems to be
- iii. No convergence, possible stability
- iv. The lower  $\beta$  is, the slower convergence seems to be
- ii) Nuemann, slower convergence
  - a. Convergence happens much faster
  - b. The smaller  $\alpha$  is, the slower the convergence seems to be
  - c. No convergence, possible stability
  - d. The lower  $\beta$  is, the slower convergence seems to be
  - e. Expo
    - i. Convergence happens much faster
    - ii. The smaller  $\alpha$  is, the slower the convergence seems to be
    - iii. No convergence, possible stability
    - iv. The lower  $\beta$  is, the slower convergence seems to be
- iii) Periodic, less wavy
  - a. Convergence happens much faster
  - b. The smaller  $\alpha$  is, the slower the convergence seems to be
  - c. No convergence, possible stability
  - d. The lower  $\beta$  is, the slower convergence seems to be
  - e. Expo
    - i. Convergence happens much faster
    - ii. The smaller  $\alpha$  is, the slower the convergence seems to be
    - iii. No convergence, possible stability
    - iv. The lower  $\beta$  is, the slower convergence seems to be
- iv) The  $\alpha$  and  $\beta$  seems to be able to tweak the speed of conversion
- v) (i) the observations didn't differ
  - a. Convergence happens much faster
  - b. The smaller  $\alpha$  is, the slower the convergence seems to be
  - c. No convergence, possible stability

d. The lower  $\boldsymbol{\beta}$  is, the slower convergence seems to be

## MatLab – Exercise 9.2.2

- i) Heat disperses more quickly, sine, Dirichlet
  - a. Now long-term behavior is similar to the heat equation
  - b. There is almost no convergence, this happens with very small k^2 in the heat equation
  - c. Expo, Dirichlet
    - i. Long=term behavior is the similar as the heat equation when  $\alpha = 0$
    - ii. There is almost no convergence, with  $\beta = 0$ , this happens in the heat equation when k^2 is very small
- ii) Fluid seepage converges slower than the heat equation under these conditions, Cos, Nuemann
  - a. Now long-term behavior is similar to the heat equation
  - b. There is almost no convergence, this happens with very small k^2 in the heat equation
  - c. Expo, Nuemann
    - i. Long=term behavior is the similar as the heat equation when  $\alpha = 0$
    - ii. There is almost no convergence, with  $\beta = 0$ , this happens in the heat equation when k<sup>2</sup> is very small
- iii) Periodic, Cos, Fluid seepage converges slower than the heat equation
  - a. Now long-term behavior is similar to the heat equation
  - b. There is almost no convergence, this happens with very small k^2 in the heat equation
  - c. Expo, Periodic
    - i. Long=term behavior is the similar as the heat equation when  $\alpha = 0$
    - ii. There is almost no convergence, with  $\beta$  = 0, this happens in the heat equation when k^2 is very small
- iv) With  $\alpha$  very small or 0, and/or  $\beta$  very large, the fluid seepage equation can converge similarly as the heat equation, with (k^2) = 1. With k^2 very small in the heat equation convergence is similar to  $\beta$  very small or 0 in the fluid seepage equation.
- v) Periodic, step, heat equation disperses more quickly
  - a. Now long-term behavior is similar to the heat equation
  - b. There is almost no convergence, this happens with very small k^2 in the heat equation
  - c. Expo, Periodic
    - i. Long=term behavior is the similar as the heat equation when  $\alpha = 0$
    - ii. There is almost no convergence, with  $\beta = 0$ , this happens in the heat equation when k^2 is very small
    - iii. This is the same as the other condition and compare to the heat equation.

MatLab – Exercise 9.2.3

- a. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
- b. Repeat
  - i. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
  - ii. 0.1 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.1
  - iii. 0.01 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.01
  - iv. 0.005 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.005
- c. Seems to be dependent on initial conditions
- d. Expo, step
  - i. Expo
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.11129
    - 2. Repeat
      - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.055645
      - b. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.022258
      - c. 0.01 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.0022258
      - d. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.0011129
    - 3. Seems to be dependent on initial conditions
  - ii. Step
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
    - 2. Repeat
      - e. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
      - f. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.1
      - g. 0.01 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.01
      - h. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.005
    - 3. Seems to be dependent on initial conditions
- e. The fluid seepage equation seems to correspond with the heat equation
- ii) Cos
  - a. 1.1 Norm Different vs. Time, Different Parameters = 0.1

Norm Different between Initial Conditions = 0

- b. Repeat with  $\alpha$  values
  - v. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - vi. 0.98 Norm Different vs. Time, Different Parameters = 0.02 Norm Different between Initial Conditions = 0.
  - vii. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- c. Seems to be dependent on initial conditions
- d. Repeat with  $\beta$  values
  - i. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - ii. 1.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0.
  - iii. 1.99 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
  - iv. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- e. Expo and step
  - i. Expo
    - 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat for  $\alpha$  values
      - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
        Norm Different between Initial Conditions = 0
      - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - 3. Seems to be dependent on initial conditions
    - 4. Repeat for  $\beta$  values
      - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - d. 1.9 Norm Different vs. Time, Different Parameters = 0.1
        Norm Different between Initial Conditions = 0
      - e. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
      - f. 2.01 Norm Different vs. Time, Different Parameters = 0.01
        Norm Different between Initial Conditions = 0
    - 5. Seems to be dependent on initial conditions
  - ii. Step
    - 1. 1.1 Norm Different vs. Time, Different Parameters = 0.1

Norm Different between Initial Conditions = 0

- 2. Repeat for  $\alpha$  values
  - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
    Norm Different between Initial Conditions = 0
  - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- 3. Seems to be dependent on initial conditions
- 4. Repeat for  $\beta$  values
  - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 1.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - c. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - d. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- 5. Seems to be dependent on initial conditions
- f. Still seems to depend on the initial conditions
- iii) Cos
  - a. Yes difference is small
  - b. Repeat for values of  $\boldsymbol{\alpha}$ 
    - i.  $\alpha = .9$  the difference seems small
    - ii.  $\alpha$  = .98 the difference seems small
    - iii.  $\alpha$  = 1.01 the difference seems small
  - c. changing  $\alpha$  does not seem to change the dependence on the initial conditions
  - d. Repeat for values of  $\beta$ 
    - i.  $\beta$  = 2.1 the difference seems small
    - ii.  $\beta = 1.9$  the difference seems small
    - iii.  $\beta = 1.99$  the difference seems small
    - iv.  $\beta = 2.01$  the difference seems small
  - e. Still seems to depend on initial conditions
  - f. Still seems to depend on initial conditions with different perturbation sizes.
- iv) In 9.3, there was a dependence on 2 variables, the initial conditions and k^2. While here, the dependence seems to be on initial conditions, initial conditions,  $\alpha$ , and  $\beta$ .
- v) Expo and step
  - a. Expo
    - i. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0.0011129
    - ii. 0.001 Norm Different vs. Time, Different Parameters = 0.001 Norm Different between Initial Conditions = 0.0011129

- iii. There seems to be a sensitivity to  $\alpha$  being very small
- b. Step
  - i. There seems to be no difference
  - ii. There seems to be no difference
  - iii. There seems to be a sensitivity to  $\alpha$  being very small

## MatLab – Exercise 9.2.4

- i. BC Dirichlet, converges slower than the Heat Equation
  - a. Similar convergence to Heat Equation with  $\alpha = 0$
  - b. The smaller  $\alpha$ , the faster convergence is
  - c. Little or no convergence with  $\beta = 0$
  - d. The smaller  $\beta$ , the slower convergence
  - e. Expo convergens is slower than the Heat Equation
    - 1. Similar convergence to Heat Equation with  $\alpha = 0$
    - 2. The smaller  $\alpha$ , the faster convergence is
    - 3. Little or no convergence with  $\beta = 0$
    - 4. The smaller  $\beta$ , the slower convergence
- ii. BC Nuemann, converges slower than the Heat Equation
  - a. Similar convergence to Heat Equation with  $\alpha = 0$
  - b. The smaller  $\alpha$ , the faster convergence is, same as (i)(b)
  - c. Little or no convergence with  $\beta = 0$ , same as (i)(c)
  - d. The smaller  $\beta$ , the slower convergence, same as (i)(d)
  - e. Expo convergence is slower than the Heat Equation
    - 1. Similar convergence to Heat Equation with  $\alpha = 0$
    - 2. The smaller  $\alpha$ , the faster convergence is, same as (i)(b)
    - 3. Little or no convergence with  $\beta = 0$ , same as (i)(c)
    - 4. The smaller  $\beta$ , the slower convergence, same as (i)(d)
- iii. BC Periodic, converges slower than the Heat Equation
  - a. Similar convergence to Heat Equation with  $\alpha = 0$
  - b. The smaller  $\alpha$ , the faster convergence is
  - c. Little or no convergence with  $\beta = 0$
  - d. The smaller  $\beta$ , the slower convergence
  - e. Expo convergens is slower than the Heat Equation
    - 1. Similar convergence to Heat Equation with  $\alpha = 0$
    - 2. The smaller  $\alpha$ , the faster convergence is
    - 3. Little or no convergence with  $\beta = 0$
    - 4. The smaller  $\beta$ , the slower convergence
- iv. The evolution model seems to be similar to Explore 9.9
- v. Step, no changes converges slower than the Heat Equation
  - a. Similar convergence to Heat Equation with  $\alpha = 0$

- b. The smaller  $\alpha$ , the faster convergence is
- c. Little or no convergence with  $\beta = 0$
- d. The smaller  $\beta$ , the slower convergence

MatLab – Exercise 9.2.5

- i. BC Dirichlet, fluid disperses moderately
  - a. When  $\alpha$  = 0, convergence is the same as the Heat Equation
  - b. Seems a little slower
  - c. Converges faster than (b), and is similar to the Heat Equation
  - d. Expo, fluid disperses moderately
    - 1. When  $\alpha = 0$ , convergence is the same as the Heat Equation
    - 2. Seems a little slower
    - 3. Converges faster than (2), and is similar to the Heat Equation
- ii. BC Nuemann, fluid disperses moderately
  - a. When  $\alpha$  = 0, convergence is the same as the Heat Equation
  - b. Seems a little slower
  - c. Converges faster than (b), and is similar to the Heat Equation
  - e. Expo, fluid disperses moderately
    - 1. When  $\alpha = 0$ , convergence is the same as the Heat Equation
    - 2. Seems a little slower
    - 3. Converges faster than (2), and is similar to the Heat Equation
- ii. BC Periodic, fluid disperses moderately
  - a. When  $\alpha = 0$ , convergence is the same as the Heat Equation
  - b. Seems a little slower
  - c. Converges faster than (b), and is similar to the Heat Equation
  - d. Expo, fluid disperses moderately
    - 1. When  $\alpha$  = 0, convergence is the same as the Heat Equation
    - 2. Seems a little slower
    - 3. Converges faster than (2), and is similar to the Heat Equation
- iii. B seems to be the dominant variable, over  $\boldsymbol{\alpha}$
- iv. Step, fluid disperses moderately
  - e. When  $\alpha$  = 0, convergence is the same as the Heat Equation
  - f. Seems a little slower
  - g. Converges faster than (b), and is similar to the Heat Equation

## MatLab – Exercise 9.2.6

vi) Sine

- a. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
- b. Repeat
  - viii. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
  - ix. 0.1 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.1
  - x. 0.01 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.01
  - xi. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.005
- c. Seems to be dependent on initial conditions
- d. Expo, step
  - i. Expo
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.099083
    - 2. Repeat
      - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.049542
      - b. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.019817
      - c. 0.01 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.0019817
      - d. 0.005 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.00099083
    - 3. Seems to be dependent on initial conditions
  - ii. Step
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
    - 2. Repeat
      - e. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
      - f. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.1
      - g. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.01
      - h. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.005
    - 3. Seems to be dependent on initial conditions
- e. The fluid seepage equation seems to correspond with the heat equation

vii) Cos

a. 1.1 - Norm Different vs. Time, Different Parameters = 0.1

Norm Different between Initial Conditions = 0

- b. Repeat with  $\alpha$  values
  - xii. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - xiii. 0.98 Norm Different vs. Time, Different Parameters = 0.02 Norm Different between Initial Conditions = 0.
  - xiv. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- c. Seems to be dependent on initial conditions
- d. Repeat with  $\beta$  values
  - i. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - ii. 1.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0.
  - iii. 1.99 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
  - iv. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- e. Expo and step
  - i. Expo
    - 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat for  $\alpha$  values
      - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
        Norm Different between Initial Conditions = 0
      - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - 3. Seems to be dependent on initial conditions
    - 4. Repeat for  $\beta$  values
      - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - d. 1.9 Norm Different vs. Time, Different Parameters = 0.1
        Norm Different between Initial Conditions = 0
      - e. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
      - f. 2.01 Norm Different vs. Time, Different Parameters = 0.01
        Norm Different between Initial Conditions = 0
    - 5. Seems to be dependent on initial conditions
  - ii. Step
    - 1. 1.1 Norm Different vs. Time, Different Parameters = 0.1

Norm Different between Initial Conditions = 0

- 2. Repeat for  $\alpha$  values
  - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
    Norm Different between Initial Conditions = 0
  - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- 3. Seems to be dependent on initial conditions
- 4. Repeat for  $\beta$  values
  - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 1.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - c. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - d. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- 5. Seems to be dependent on initial conditions
- f. Still seems to depend on the initial conditions

viii) Cos

- a. Yes difference is small
- b. Repeat for values of  $\boldsymbol{\alpha}$ 
  - i.  $\alpha = .9$  the difference seems small
  - ii.  $\alpha$  = .98 the difference seems small
  - iii.  $\alpha = 1.01$  the difference seems small
- c. changing  $\alpha$  does not seem to change the dependence on the initial conditions
- d. Repeat for values of  $\beta$ 
  - i.  $\beta$  = 2.1 the difference seems small
  - ii.  $\beta = 1.9$  the difference seems small
  - iii.  $\beta = 1.99$  the difference seems small
  - iv.  $\beta = 2.01$  the difference seems small
- e. Still seems to depend on initial conditions
- f. Still seems to depend on initial conditions with different perturbation sizes.
- ix) In 9.3, there was a dependence on 2 variables, the initial conditions and k^2. While here, the dependence seems to be on initial conditions, initial conditions,  $\alpha$ , and  $\beta$ .
- x) Expo and step
  - a. Expo
    - i. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0.0011129
    - ii. 0.001 Norm Different vs. Time, Different Parameters = 0.001 Norm Different between Initial Conditions = 0.0011129

iii. There seems to be a sensitivity to  $\alpha$  being very small

## b. Step

- i. There seems to be no difference
- ii. There seems to be no difference
- iii. There seems to be a sensitivity to  $\boldsymbol{\alpha}$  being very small

## MatLab – Exercise 9.2.7

- i. Nuemann, cos. Convergence seems similar to the fluid seepage equation, may be a little slower.
  - a. Glitch
  - b. The smaller  $\alpha$ , the less the equation converges
  - c. Little or no convergence
  - d. The smaller  $\beta$ , the slower convergence
  - e. Expo, convergence seems similar to the fluid seepage equation, may be a little slower
    - i. Glitch
    - ii. The smaller  $\alpha$ , the less the equation converges
    - iii. Little or no convergence
    - iv. The smaller  $\beta$ , the slower convergence
- ii. Periodic, cos. Convergence seems similar to the fluid seepage equation, may be a little slower.
  - a. Glitch
  - b. The smaller  $\alpha$ , the less the equation converges
  - c. Little or no convergence
  - d. The smaller  $\beta$ , the slower convergence
  - e. Expo, convergence seems similar to the fluid seepage equation, may be a little slower
    - i. Glitch
    - ii. The smaller  $\boldsymbol{\alpha}$  , the less the equation converges
    - iii. Little or no convergence
    - iv. The smaller  $\beta$ , the slower convergence
- iii. B seems to be the dominant variable, and it seems to be similar to the fluid seepage equation.
- iv. Step, convergence seems similar to the fluid seepage equation, may be a little slower.
  - a. Glitch
  - b. The smaller  $\alpha$ , the less the equation converges
  - c. Little or no convergence
  - d. The smaller  $\beta$ , the slower convergence

## MatLab – Exercise 9.2.8

- 1. Nuemann, cos, relatively slow convergence, slower than fluid seepage equation
  - a. Glitch
  - b. Little or no convergence
  - c. Expo, relatively slow convergence, slower than fluid seepage equation
    - i. Glitch
    - ii. Little or no convergence
- 2. Periodic, cos, relatively slow convergence, slower than fluid seepage equation
  - a. Glitch
  - b. Little or no convergence
  - c. Expo, relatively slow convergence, slower than fluid seepage equation
    - i. Glitch
    - ii. Little or no convergence
- 3. The smaller  $\alpha$ , the faster convergence, and the smaller  $\beta$ , the slower convergence. With  $\beta$  being the dominant variable. Similar to Explore! 9.9.
- 4. Periodic, step, relatively slow convergence, slower than fluid seepage equation
  - a. Glitch
  - b. Little or no convergence
  - c. No changes need to be made to (3)

## <mark>MatLab – Exercise 9.2.9</mark>

- xi) Neumann, cos
  - a. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
  - b. Repeat
    - xv. 0.25 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.25

- xvi. 0.1 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.1
- xvii. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.01
- xviii. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.005
- c. Seems to be dependent on initial conditions
- d. Expo, step
  - i. Expo
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.099083
    - 2. Repeat
      - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.049542
      - b. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.019817
      - c. 0.01 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.0019817
      - d. 0.005 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.00099083
    - 3. Seems to be dependent on initial conditions
  - ii. Step
    - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
    - 2. Repeat
      - e. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
      - f. 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.1
      - g. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.01
      - h. 0.005 Norm Different vs. Time, Different Parameters = 0
        - Norm Different between Initial Conditions = 0.005
    - 3. Seems to be dependent on initial conditions
- e. The fluid seepage equation seems to correspond with the heat equation
- xii) Cos
  - a. 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. Repeat with  $\alpha$  values
    - xix. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - xx. 0.98 Norm Different vs. Time, Different Parameters = 0.02

Norm Different between Initial Conditions = 0.

- xxi. 1.01 Norm Different vs. Time, Different Parameters = 0.01
  - Norm Different between Initial Conditions = 0
- c. Seems to be dependent on initial conditions
- d. Repeat with  $\beta$  values
  - i. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - ii. 1.9 Norm Different vs. Time, Different Parameters = 0.1Norm Different between Initial Conditions = 0.
  - iii. 1.99 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
  - iv. 2.01 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
- e. Expo and step
  - i. Expo
    - 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat for  $\alpha$  values
      - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
        Norm Different between Initial Conditions = 0
      - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - 3. Seems to be dependent on initial conditions
    - 4. Repeat for  $\beta$  values
      - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - d. 1.9 Norm Different vs. Time, Different Parameters = 0.1
        Norm Different between Initial Conditions = 0
      - e. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
      - f. 2.01 Norm Different vs. Time, Different Parameters = 0.01
        Norm Different between Initial Conditions = 0
    - 5. Seems to be dependent on initial conditions
  - ii. Step
    - 1. 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat for  $\alpha$  values
      - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02

Norm Different between Initial Conditions = 0

- c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- 3. Seems to be dependent on initial conditions
- 4. Repeat for  $\beta$  values
  - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 1.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - c. 1.99 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
  - d. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- 5. Seems to be dependent on initial conditions
- f. Still seems to depend on the initial conditions

### xiii) Cos

- a. Yes difference is small
- b. Repeat for values of  $\boldsymbol{\alpha}$ 
  - i.  $\alpha$  = .9 the difference seems small
  - ii.  $\alpha$  = .98 the difference seems small
  - iii.  $\alpha$  = 1.01 the difference seems small
- c. changing  $\alpha$  does not seem to change the dependence on the initial conditions
- d. Repeat for values of  $\beta$ 
  - i.  $\beta = 2.1$  the difference seems small
  - ii.  $\beta = 1.9$  the difference seems small
  - iii.  $\beta = 1.99$  the difference seems small
  - iv.  $\beta = 2.01$  the difference seems small
- e. Still seems to depend on initial conditions
- f. Still seems to depend on initial conditions with different perturbation sizes.

### MatLab – Exercise 9.2.10

- i. Nuemann, cos, convergence is similar to the 2D fluid seepage equation, maybe a little slower
  - a. Glitch
  - b. The smaller  $\alpha$  is, the faster convergence seems to be
  - c. Little or no convergence
  - d. The smaller  $\beta$  is, the slower convergence
  - e. Expo
    - i. Glitch
    - ii. The smaller  $\alpha$  is, the faster convergence seems to be

- iii. Little or no convergence
- iv. The smaller  $\boldsymbol{\beta}$  is, the slower convergence
- ii. Periodic, cos, cos, convergence is similar to the 2D fluid seepage equation, maybe a little slower
  - a. Glitch
  - b. The smaller  $\alpha$  is, the faster convergence seems to be
  - c. Little or no convergence
  - d. The smaller  $\boldsymbol{\beta}$  is, the slower convergence
  - e. Expo
    - i. Glitch
    - ii. The smaller  $\boldsymbol{\alpha}$  is, the faster convergence seems to be
    - iii. Little or no convergence
    - iv. The smaller  $\beta$  is, the slower convergence
- iii. B is the dominant variable, similar to Explore! 9.9
- iv. Step, convergence is similar to the 2D fluid seepage equation, maybe a little slower
  - a. Glitch
  - b. The smaller  $\alpha$  is, the faster convergence seems to be
  - c. Little or no convergence
  - d. The smaller  $\boldsymbol{\beta}$  is, the slower convergence
  - e. No changes to (iii) are needed

## MatLab – Exercise 9.2.11

- i. No sine initial condition option
  - a. No sine initial condition option
  - b. No sine initial condition option
  - c. Expo, convergence is similar to the heat equation, but much slower
    - i. Glitch
    - ii. Little or no convergence
- ii. Periodic, cos convergence is similar to the heat equation, but slower
  - a. Glitch
  - b. Little or no convergence
  - c. Expo, convergence is similar to the heat equation, but much slower
    - i. Glitch
    - ii. Little or no convergence
- iii. Summary is the same as Explore! 9.15
- iv. Step, convergence is similar to the heat equation, but much slower
  - a. Glitch

- b. Little or no convergence
- c. No difference in observations

### MatLab – Exercise 9.2.12

xiv) Cos

- a. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 1.4142
- b. Repeat
  - xxii. 0.25 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.70711
  - xxiii. 0.1 Norm Different vs. Time, Different Parameters = 0
    - Norm Different between Initial Conditions = 0.28284
  - xxiv. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.028284
  - xxv. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.014142
- c. Seems to be dependent on initial conditions
- d. Expo, step
  - i. Expo

 0.5 - Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.099083

- 2. Repeat
  - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.049542
  - b. 0.1 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.019817
  - c. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.0019817
  - d. 0.005 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.00099083
- 3. Seems to be dependent on initial conditions
- ii. Step
  - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 31
  - 2. Repeat

- e. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 15.5
- f. 0.1 Norm Different vs. Time, Different Parameters = 0
  Norm Different between Initial Conditions = 6.2
- g. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.62
- h. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.31
- 3. Seems to be dependent on initial conditions
- e. The fluid seepage equation seems to correspond with the heat equation

xv) Cos

- a. 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
- b. Repeat with  $\alpha$  values
  - xxvi. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - xxvii. 0.98 Norm Different vs. Time, Different Parameters = 0.02 Norm Different between Initial Conditions = 0.
  - xxviii. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- c. Seems to be dependent on initial conditions
- d. Repeat with  $\beta$  values
  - i. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - ii. 1.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0.
  - iii. 1.99 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
  - iv. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    - Norm Different between Initial Conditions = 0
- e. Expo and step
  - і. Ехро
    - 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat for  $\alpha$  values
      - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
      - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
        Norm Different between Initial Conditions = 0
      - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - 3. Seems to be dependent on initial conditions

- 4. Repeat for  $\beta$  values
  - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - d. 1.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - e. 1.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - f. 2.01 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- 5. Seems to be dependent on initial conditions
- ii. Step
  - 1.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - 2. Repeat for  $\alpha$  values
    - a. 0.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - b. 0.98 Norm Different vs. Time, Different Parameters = 0.02
      Norm Different between Initial Conditions = 0
    - c. 1.01 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
  - 3. Seems to be dependent on initial conditions
  - 4. Repeat for  $\beta$  values
    - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - b. 1.9 Norm Different vs. Time, Different Parameters = 0.1
      Norm Different between Initial Conditions = 0
    - c. 1.99 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
    - d. 2.01 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
  - 5. Seems to be dependent on initial conditions
- f. Still seems to depend on the initial conditions

xvi) Cos

- a. Yes difference is small
- b. Repeat for values of  $\alpha$ 
  - i.  $\alpha$  = .9 the difference seems small
  - ii.  $\alpha$  = .98 the difference seems small
  - iii.  $\alpha = 1.01$  the difference seems small
- c. changing  $\alpha$  does not seem to change the dependence on the initial conditions
- d. Repeat for values of  $\boldsymbol{\beta}$ 
  - i.  $\beta = 2.1$  the difference seems small
  - ii.  $\beta = 1.9$  the difference seems small

- iii.  $\beta$  = 1.99 the difference seems small
- iv.  $\beta = 2.01$  the difference seems small
- e. Still seems to depend on initial conditions
- f. Still seems to depend on initial conditions with different perturbation sizes.
- xvii) Can't do this exercise

## MatLab – Exercise 9.3.1

- i) Dirichlet sine, runs
  - a. Runs
  - b. Motion is slower
  - c. Runs
  - d. The smaller c^2, the slower the motion
  - e. The larger c^2, the faster motion is
- ii) Runs
  - a. Runs
  - b. Motion is slower
  - c. Runs
  - d. The smaller c^2, the slower the motion
  - e. The larger c^2, the faster motion is
- iii) Runs
  - a. Runs
  - b. Motion is slower
  - c. Runs
  - d. The smaller c^2, the slower the motion
  - e. The larger c^2, the faster motion is
- iv) The Heat equation would converge to zero relatively quickly, but the wave equation does not

## MatLab – Exercise 9.3.2

- i. There's a series of ripples
  - a. Less time between waves
  - b. There's a series of peaks and troughs
    - i. Not much change
- ii. Repeat
  - a. Cos, no pattern
    - i. Smaller waves, still no pattern
  - b. Step, argil pattern
    - i. Denser pattern
- iii. C^2 = 3
  - a. Cos
    - i. no pattern
    - ii. No difference
  - b. Sine
    - i. Series of ripples
    - ii. Less time between waves
  - c. Expo
    - i. X patterns of peaks and troughs
    - ii. No difference
  - d. Step
    - i. Argil pattern
    - ii. Denser pattern
- iv. The heat equation dissipates and stops. While the wave equation continues on much longer
- v. Dirichlet is more fluid, while the other 2 are more ridged

## Matlab – Exercise 9.3.3

- i. Sin, 0
  - a. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5
  - b. Repeat

xxix. 0.25 - Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25

- xxx. 0.1 Norm Different vs. Time, Different Parameters = 0
  - Norm Different between Initial Conditions = 0.1
- xxxi. 0.01 Norm Different vs. Time, Different Parameters = 0
  - Norm Different between Initial Conditions = 0.01
- xxxii. 0.005 Norm Different vs. Time, Different Parameters = 0
  - Norm Different between Initial Conditions = 0.005
- xxxiii. Wave equation seems to depend on initial conditions
- c. Non-zero, and expo
  - i. Sine, 1

1. 0.5 - Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.5

- 2. Repeat
  - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
  - b. 0.1 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.1
  - c. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.01
  - d. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.005
- ii. Expo, 0
  - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.22258
  - 2. Repeat
    - e. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.11129
    - f. 0.1 Norm Different vs. Time, Different Parameters = 0
      Norm Different between Initial Conditions = 0.044516
    - g. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.0044516
    - h. 0.005 Norm Different vs. Time, Different Parameters = 0
      - Norm Different between Initial Conditions = 0.022258
- iii. Expo, 1
  - 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.2258
  - 2. Repeat
    - i. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.11129
    - j. 0.1 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.044516
    - k. 0.01 Norm Different vs. Time, Different Parameters = 0
      Norm Different between Initial Conditions = 0.0044516

- I. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.0022258
- a. No change to conjecture
- d. Perturbing c<sup>2</sup>, d and e
  - i. Cos, 0
    - 3.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - 2. Repeat
      - m. 2.9 Norm Different vs. Time, Different Parameters = 0.1
        Norm Different between Initial Conditions = 0
      - n. 2.95 Norm Different vs. Time, Different Parameters = 0.05 Norm Different between Initial Conditions = 0
      - o. 2.99 Norm Different vs. Time, Different Parameters = 0.01
        Norm Different between Initial Conditions = 0
      - p. 3.1 Norm Different vs. Time, Different Parameters = 0.01
        Norm Different between Initial Conditions = 0
- e. Repeat d with non-zero and expo, this is f
  - i. Cos,1

 3.1 - Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0

- 2. Repeat
  - q. 2.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - r. 2.95 Norm Different vs. Time, Different Parameters = 0.05
    Norm Different between Initial Conditions = 0
  - s. 2.99 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - t. 3.1 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- ii. Expo, O
  - 3.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - 2. Repeat
    - u. 2.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - v. 2.95 Norm Different vs. Time, Different Parameters = 0.05 Norm Different between Initial Conditions = 0
    - w. 2.99 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
    - x. 3.1 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
- iii. Expo, 1

- 3.1 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
- 2. Repeat
  - y. 2.9 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
  - 2.95 Norm Different vs. Time, Different Parameters = 0.05
    Norm Different between Initial Conditions = 0
  - aa. 2.99 Norm Different vs. Time, Different Parameters = 0.01Norm Different between Initial Conditions = 0
  - bb. 3.1 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- f. This is really g and h
  - i. Cos, 0
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
    - 0.5 Norm Difference between Initial Conditions = 0.70711
    - 2. Repeat
      - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 0.14142
      - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
        0.051 Norm Difference between Initial Conditions = 0.070711
      - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01 0.01 - Norm Difference between Initial Conditions = 0.014142
  - ii. Cos, 1
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
    - 0.5 Norm Difference between Initial Conditions = 0.70711
    - 2. Repeat
      - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 0.14142
      - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
        0.051 Norm Difference between Initial Conditions = 0.070711
      - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01 0.01 - Norm Difference between Initial Conditions = 0.014142
  - iii. Expo, 0
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.5 Norm Difference between Initial Conditions = 0.22258
    - 2. Repeat
      - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 0.022258
      - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
        - 0.01 Norm Difference between Initial Conditions = 0.0044516
      - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01 0.01 - Norm Difference between Initial Conditions = 0.0044516
  - iv. Expo, 1
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.5 Norm Difference between Initial Conditions = 0.22258

- 2. Repeat
  - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 0.022258
  - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01 0.051 - Norm Difference between Initial Conditions = 0.0044516
  - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01
    - 0.01 Norm Difference between Initial Conditions = 0.004451

- ii. Skip
- iii. Skip
- iv. Repeat (i) with H<sup>1</sup>, perturbation size
  - a. Sin, O
    - v. 0.5 Norm Different vs. Time, Different Parameters = 0
      - Norm Different between Initial Conditions = 1.6485
    - vi. Repeat
      - 0.25 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.824223
      - 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.32969
      - 3. 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.032969
      - 4. 0.005 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.016485
  - b. Sin, 1
    - vii. 0.5 Norm Different vs. Time, Different Parameters = 0
      - Norm Different between Initial Conditions = 1.6485
    - viii. Repeat
      - 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.824223
      - 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.32969
      - 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.032969
      - 0.005 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.016485
  - c. Expo, 0
    - ix. 0.5 Norm Different vs. Time, Different Parameters = 0
      - Norm Different between Initial Conditions = 1.4252
    - x. Repeat
      - 0.25 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.7126
      - 0.1 Norm Different vs. Time, Different Parameters = 0
        Norm Different between Initial Conditions = 0.28504

- 0.01 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.028504
- 0.005 Norm Different vs. Time, Different Parameters = 0
  Norm Different between Initial Conditions = 0.014252
- d. Expo, 1
  - xi. 0.5 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 1.4252
  - xii. Repeat
    - 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.7126
    - 0.1 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.28504
    - 0.01 Norm Different vs. Time, Different Parameters = 0
      Norm Different between Initial Conditions = 0.028504
    - 0.005 Norm Different vs. Time, Different Parameters = 0
      Norm Different between Initial Conditions = 0.014252
- e. C^2 changes

i. Cos, 0

 0.5 - Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0

- 2. Repeat
  - a. 0.25 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 0.1 Norm Different vs. Time, Different Parameters = 0.05 Norm Different between Initial Conditions = 0
  - c. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - d. 0.005 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- ii. Cos, 1
  - 0.5 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - 4. Repeat
    - a. 0.25 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - b. 0.1 Norm Different vs. Time, Different Parameters = 0.05
      Norm Different between Initial Conditions = 0
    - c. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - d. 0.005 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
- iii. Expo, 0

- 0.5 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
- 6. Repeat
  - a. 0.25 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - b. 0.1 Norm Different vs. Time, Different Parameters = 0.05
    Norm Different between Initial Conditions = 0
  - c. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
  - d. 0.005 Norm Different vs. Time, Different Parameters = 0.01
    Norm Different between Initial Conditions = 0
- iv. Expo, 1
  - 0.5 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
  - 8. Repeat
    - a. 0.25 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0
    - b. 0.1 Norm Different vs. Time, Different Parameters = 0.05
      Norm Different between Initial Conditions = 0
    - c. 0.01 Norm Different vs. Time, Different Parameters = 0.01 Norm Different between Initial Conditions = 0
    - d. 0.005 Norm Different vs. Time, Different Parameters = 0.01
      Norm Different between Initial Conditions = 0
- f. This is g and h
  - i. Cos, 0
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.5 Norm Difference between Initial Conditions = 6.5155
    - 2. Repeat
      - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 1.3031
      - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
        0.01 Norm Difference between Initial Conditions = 0.65155
      - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01
        - 0.01 Norm Difference between Initial Conditions = 0.13031
  - ii. Cos, 1
    - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
      - 0.5 Norm Difference between Initial Conditions = 6.5155
    - 2. Repeat
      - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions = 1.3031
      - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
        0.01 Norm Difference between Initial Conditions = 0.65155
      - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01
        0.01 Norm Difference between Initial Conditions = 0.13031

- iii. Expo, 0
  - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
    - 0.5 Norm Difference between Initial Conditions = 1.4252
  - 2. Repeat
    - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions =0.28504
    - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
      0.01 Norm Difference between Initial Conditions = 0.14252
    - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01 0.01 - Norm Difference between Initial Conditions = 0.028504
- iv. Expo, 1
  - 1. 3.1 Norm Difference vs. Time Difference in Parameters = 0.1
    - 0.5 Norm Difference between Initial Conditions = 1.4252
  - 2. Repeat
    - a. 2.95 Norm Difference vs. Time Difference in Parameters = 0.05 0.1 - Norm Difference between Initial Conditions =0.28504
    - b. 2.99 Norm Difference vs. Time Difference in Parameters = 0.01
      0.01 Norm Difference between Initial Conditions = 0.14252
    - c. 3.01 Norm Difference vs. Time Difference in Parameters = 0.01 0.01 - Norm Difference between Initial Conditions = 0.028504

v. skip

#### MatLab – Exercise 9.3.4

- i. Runs
  - a. Runs
  - b. They seem relatively similar
  - c. They seem relatively similar
  - d. A larger  $\alpha$  yields less fluctuation
  - e. The larger  $\alpha$ , the larger the wave fluctuation

#### ii. Runs

- a. Runs
- b. They seem relatively similar
- c. They seem relatively similar
- d. A larger  $\alpha$  yields less fluctuation
- e. The larger  $\alpha$ , the larger the wave fluctuation
- iii. Runs

- a. Runs
- b. They seem relatively similar
- c. They seem relatively similar
- d. A larger  $\alpha$  yields less fluctuation
- e. The larger  $\alpha$ , the larger the wave fluctuation

### MatLab – Exercise 9.3.5

- i. They get smaller as time goes by
  - a. Slower decline in wave fluctuation
  - b. Slower decline in wave fluctuation
- ii. Damped slows rapidly
  - a. Damped slows rapidly
  - b. Damped slows rapidly
- iii. Damped slows rapidly
  - a. Damped slows rapidly
  - b. Damped slows rapidly
- iv. The smaller  $\alpha$ , the smaller the dampening effects are
- v. The conjecture from (iv) holds
- vi. The lower  $c^2$ , the greater the difference between damped and undamped.

## ii. Sin, 0

- g. 0.5 Norm Different vs. Time, Different Parameters = 0
  Norm Different between Initial Conditions = 0.5
- h. Repeat
  - i. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 0.25
  - ii. 0.1 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.1
  - iii. 0.01 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.01
  - iv. 0.005 Norm Different vs. Time, Different Parameters = 0
    Norm Different between Initial Conditions = 0.005
  - v. Seems
- i. Conjecture holds for non-zero initial velocity
- iii. Cos, 0
  - j. P.p.c 3.1 Norm Different vs. Time, Different Parameters = 0.14142 p.d.c. - 4.9 - Norm Different between Initial Conditions = 0
  - k. Repeat
    - P.p.c 2.9 Norm Different vs. Time, Different Parameters = 0.14142
      p.d.c. 5.1 Norm Different between Initial Conditions = 0
    - P.p.c 2.95 Norm Different vs. Time, Different Parameters = 0.070711
      p.d.c. 4.95 Norm Different between Initial Conditions = 0
    - P.p.c 2.99 Norm Different vs. Time, Different Parameters = 0.05099
      p.d.c. 5.05 Norm Different between Initial Conditions = 0
    - iv. P.p.c 3.01 Norm Different vs. Time, Different Parameters = 0.014142
      - p.d.c. 4.99 Norm Different between Initial Conditions = 0
    - v. Purterbing  $\alpha$  and  $c^2$ , seems to change the model a bit
  - I. Conjecture holds for non-zero initial velocity
- iv. 9.2.3 had no dampening effects, it was fluid

## MatLab – Exercise 9.3.7

- vi. Runs
  - a. Runs
  - b. Slower, same as EXPLORE! 9.21
  - c. Slower, same as EXPLORE! 9.21
  - d. The c<sup>2</sup> the less damping affects there are with non-zero initial velocity, the waves seem to mesh together and then reform
  - e. A larger c<sup>2</sup> seems to accelerate the wave

### vii. Runs

- a. Runs
- b. Slower, same as EXPLORE! 9.21
- c. Slower, same as EXPLORE! 9.21
- d. The c<sup>2</sup> the less damping affects there are with non-zero initial velocity, the waves seem to mesh together and then reform

#### viii. Runs

- a. Runs
- b. Slower, same as EXPLORE! 9.21
- c. Slower, same as EXPLORE! 9.21
- d. The c<sup>2</sup> the less damping affects there are with non-zero initial velocity, the waves seem to mesh together and then reform

e.

## MatLab – Exercise 9.3.8

- i. 2 rows of waves
  - a. No change, 2 rows of waves
  - b. A spike that turns into a star pattern
    - i. No change, a spike that turns into a star pattern

### ii. Repeat

- a. Cos, one row of waves that breaks down with no change
- iii. repeat
  - a. 2 rows of waves

- i. No change, 2 rows of waves
- ii. A spike that turns into a star pattern
  - 1. No change, a spike that turns into a star pattern
- b. Repeat
  - i. Cos, one row of waves that breaks down with no change as initial velocity exponential
- iv. 1D patterns did not present themselves in 2D equation
- v. Step pillar that turns into a star pattern with no change as initial velocity exponential

MatLab – Exercise 9.3.9

- vi.  $C^2 = 3$ , ant t = 0.5 (sin, 0), H<sub>1</sub>
  - a. 0.25 Norm Different vs. Time, Different Parameters = 0 Norm Different between Initial Conditions = 1.7739
  - b. Repeat
    - i. 0.1 Norm Different vs. Time, Different Parameters = 0
      Norm Different between Initial Conditions = 0.70956
    - ii. 0.01 Norm Different vs. Time, Different Parameters = 0Norm Different between Initial Conditions = 0.070956
    - iii. 0.005 Norm Different vs. Time, Different Parameters = 0
      - Norm Different between Initial Conditions = 0.035478
    - iv. Seems to depend on initial conditions
  - c. No change in conjecture
- vii. (Cos, 0), with perturbation size = 0
  - a. 2.1 Norm Different vs. Time, Different Parameters = 0.1
    Norm Different between Initial Conditions = 0
    The perturbed solution is less wavy
  - b. Repeat
    - i. 1.9 Norm Different vs. Time, Different Parameters = 0.1
      Norm Different between Initial Conditions = 0
      The perturbed solution is less wavy

- ii. 1.95 Norm Different vs. Time, Different Parameters = 0.05
  Norm Different between Initial Conditions = 0
  The perturbed solution is less wavy
- iii. 2.005 Norm Different vs. Time, Different Parameters = 0.005
  Norm Different between Initial Conditions = 0
  The perturbed solution is less wavy
- c. No change in conjecture

viii. (Cos, 0) periodic boundary conditions with perturbation size = 0

- a. 2.1 Norm Different vs. Time, Different Parameters = 0.1
  Norm Different between Initial Conditions = 0
  The perturbed solution is less wavy
- b. Repeat
  - i. 1.9 Norm Different vs. Time, Different Parameters = 0.1 Norm Different between Initial Conditions = 0 The perturbed solution is less wavy
  - ii. 1.95 Norm Different vs. Time, Different Parameters = 0.05
    Norm Different between Initial Conditions = 0
    The perturbed solution is less wavy
  - iii. 2.005 Norm Different vs. Time, Different Parameters = 0.005
    Norm Different between Initial Conditions = 0
    The perturbed solution is less wavy
- c. No change in conjecture
- ix. 2D seems to depend on initial conditions, over boundary conditions and initial velocity, similar to the 1D wave equation.